Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Paper 1

	Solution	Marks	Remarks
1.	$\frac{5}{h+k}=\frac{k}{h-3}$		
	$5(h-3)=k(h+k)$	1M	
	$5h-15=hk+k^2$		
	$5h-hk=15+k^2$	1M	for putting h on one side
	$h=\frac{15+k^2}{5-k}$	1A	or equivalent
	$\frac{5}{h+k}=\frac{k}{h-3}$		
	$\frac{h+k}{5}=\frac{h-3}{k}$	1M	
	$\frac{h}{5}-\frac{h}{k}=\frac{-3}{k}-\frac{k}{5}$	1M	for putting h on one side
	$h\left(\frac{1}{5}-\frac{1}{k}\right)=\frac{-3}{k}-\frac{k}{5}$		
	$h\left(\frac{k-5}{5k}\right)=\frac{-15-k^2}{5k}$		
	$h=\frac{15+k^2}{5-k}$	1A	or equivalent
		----------(3)	
2.	$\frac{x^{-8}y}{(x^7y^9)^{-6}}$		
	$=\frac{x^{-8}y}{x^{-42}y^{-54}}$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^l=a^lb^l$
	$=x^{-8-(-42)}y^{1-(-54)}$	1M	for $\frac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\frac{1}{d^r}$
	$=x^{34}y^{55}$	1A	
		----------(3)	
3.	The least possible total weight of 250 *regular* packets of cheese		
	$=(0.22-0.005)(250)$	1M+1M	
	$=53.75$ kg		
	>53.65 kg		
	Thus, the claim is not correct.	1A	f.t.
	Note that		
	$\frac{53\,600+50}{250}$	1M+1M	
	$=214.6$ g		
	<215 g		
	Thus, the claim is not correct.	1A	f.t.
	Note that		
	$\frac{53\,600+50}{220-5}$	1M+1M	
	≈ 249.5348837		
	<250		
	Thus, the claim is not correct.	1A	f.t.
		----------(3)	

Solution	Marks	Remarks
4. (a) $3x+2>\frac{4x-5}{2}$		
$6x+4>4x-5$		
$6x-4x>-5-4$	1M	for putting x on one side
$2x>-9$		
$x>\frac{-9}{2}$	1A	$x>-4.5$
$3x-2<7$		
$x<3$		
Thus, we have $\frac{-9}{2}<x<3$.	1A	
(b) 4	1A	
	----------(4)	
5. Let x and y be the number of male passengers and the original number of female passengers on the ferry respectively.		
$\begin{cases} y=(1+40\%)x \\ x=(1+40\%)(y-24) \end{cases}$	1A+1A	
So, we have $x=1.4(1.4x-24)$.	1M	for getting a linear equation in one unknown
Solving, we have $x=35$.	1A	
Thus, the number of male passengers on the ferry is 35 .		
Let x be the number of male passengers on the ferry.		
So, the original number of female passengers on the ferry is $(1+40\%)x$.	1A	
$x=(1+40\%)((1+40\%)x-24)$	1M+1A	1M for getting a linear equation in one unknown
$x=1.4(1.4x-24)$		
Solving, we have $x=35$.	1A	
Thus, the number of male passengers on the ferry is 35 .		
The number of male passengers on the ferry		
$=\frac{(1+40\%)24}{(1+40\%)^2-1}$	1M+1A+1A	1M for fraction + 1A for numerator + 1A for denominator
$=35$	1A	
	----------(4)	

Solution	Marks	Remarks
6. (a) $a:b$ $=6:7$		
$\frac{4a-3c}{2b-c}=9$ $4a-3c=18b-9c$		
$\frac{24}{7}b-3c=18b-9c$	1M	
$c=\frac{17}{7}b$ $b:c$ $=7:17$		
$a:b:c$ $=6:7:17$	1A	
(b) Let $a=6k$, $b=7k$ and $c=17k$, where k is a non-zero constant.		
$\frac{5a+8b}{7b+3c}$ $=\frac{5(6k)+8(7k)}{7(7k)+3(17k)}$	1M	
$=\frac{86}{100}$		
$=\frac{43}{50}$	1A	0.86
	----------(4)	
7. $\angle RPS$ $=\angle PTQ-\angle PSQ$	1M	
$=68°-41°$ $=27°$		
$\angle RQS$ $=\angle RPS$ $=27°$	1A	
Note that $\angle PQR=90°$.	1M	
$\angle PQS$ $=\angle PQR-\angle RQS$ $=90°-27°$ $=63°$	1A	
	----------(4)	

Solution	Marks	Remarks
8. (a) $\angle CAE = \angle DBE$ (alt. $\angle$s, $AC // DB$)		
$\angle AEC = \angle BED$ (vert. opp. $\angle$s)		
$\angle ACE = \angle BDE$ ($\angle$ sum of Δ)		
$\Delta ACE \sim \Delta BDE$ (AAA)		(AA) (equiangular)
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(b) $\frac{BE}{BD} = \frac{AE}{AC}$ (by (a))	1M	
$\frac{20 - AE}{15} = \frac{AE}{10}$		
$AE = 8$ cm		
Note that $AC > AE > CE$.		
AC^2		
$= 10^2$		
$= 100$		
$AE^2 + CE^2$		
$= 8^2 + 7^2$		
$= 113$		
Therefore, we have $AE^2 + CE^2 \neq AC^2$.	1M	
Hence, ΔACE is not a right-angled triangle.		
By (a), ΔBDE is not a right-angled triangle.	1A	f.t.
	----------(5)	
9. (a) $49 - (20 + a) = 27$	1M	
$a = 2$		
The mean		
$= 36$	1A	
The mode		
$= 33$	1A	
(b) The required probability		
$= \frac{14}{24}$	1M	for denominator
$= \frac{7}{12}$	1A	r.t. 0.583
	----------(5)	

Solution	Marks	Remarks
10. (a) Γ is the perpendicular bisector of AB .	1M ----------(1)	
(b) (i) The slope of Γ $= -3$ The slope of AB $= \frac{1}{3}$ The equation of AB is $y+4 = \frac{1}{3}(x-2)$	1M	
$x-3y-14=0$	1A	or equivalent
(ii) Note that the centre of the required circle is the point of intersection of AB and Γ . Solving $3x+y-12=0$ and $x-3y-14=0$, the coordinates of the centre of the required circle are $(5, -3)$.	1M	
The radius of the required circle $= \sqrt{(2-5)^2+(-4+3)^2}$ $= \sqrt{10}$	1M	
Thus, the equation of the required circle is $(x-5)^2+(y+3)^2=10$.	1A	$x^2+y^2-10x+6y+24=0$
Let (h, k) be the coordinates of B . $h-3k-14=0$ $h=3k+14$ Note that the mid-point of AB lies on Γ . $3\left(\frac{2+3k+14}{2}\right)+\frac{-4+k}{2}-12=0$ Solving, we have $k=-2$ and $h=8$. Therefore, the coordinates of B are $(8, -2)$. The coordinates of the centre of the required circle $= \left(\frac{2+8}{2}, \frac{-4-2}{2}\right)$ $= (5, -3)$	1M	
The radius of the required circle $= \sqrt{(2-5)^2+(-4+3)^2}$ $= \sqrt{10}$	1M	
Thus, the equation of the required circle is $(x-5)^2+(y+3)^2=10$.	1A ----------(5)	$x^2+y^2-10x+6y+24=0$

Solution	Marks	Remarks
11. (a) $\frac{(1)(8)+(2)(5)+(3)(n)+(4)(1)}{8+5+n+1}=2$	1M	
$n=6$		
The median		
$=2$	1A	
The inter-quartile range		
$=3-1$	1M	
$=2$	1A	
The variance		
$=0.9$	1A	
	----------(5)	
(b) Note that the original range is 3 .		
There are two cases.		
Case 1: Each of the two students owns 2 calculators.		
The range of the distribution		
$=3$	1M	either one
Case 2: One of the two students owns 1 calculator and the other student owns 3 calculators.		
The range of the distribution		
$=3$		
Thus, there is no change in the range of the distribution due to the withdrawal of the two students.	1A	f.t.
	----------(2)	
12. (a) Let $f(x)=px^2+q$, where p and q are non-zero constants.	1M	
So, we have $100p+q=62$ and $225p+q=122$.	1M	for either substitution
Solving, we have $p=\frac{12}{25}$ and $q=14$.		
Hence, we have $f(x)=\frac{12}{25}x^2+14$.		
Thus, we have $f(5)=26$.	1A	
	----------(3)	
(b) By (a), we have $u=14$ and $v=26$.	1M	for either one
So, we have $UW=12$ and $VW=5$.		
Note that $\angle UWV=90°$.		
Hence, UV is a diameter of C .	1M	
The circumference of C		
$=\pi\sqrt{UW^2+VW^2}$	1M	
$=\pi\sqrt{12^2+5^2}$		
$=13\pi$	1A	
	----------(4)	

Solution	Marks	Remarks
13. (a) Let $mx+n$ be the required quotient, where m and n are constants.	1M	
Then, we have $h(x)=(mx+n)(x^3+5x^2-12x-1)+mx+n$.		
By comparing the coefficients of x^4 , we have $m=3$.	1M	either one
By comparing the coefficients of x^2 , we have $5n-12m=-16$.		
So, we have $n=4$.		
Thus, the required quotient is $3x+4$.	1A	
	----------(3)	
(b) $h(x)=0$		
$(3x+4)(x^3+5x^2-12x-1)+3x+4=0$ (by (a))		
$(3x+4)(x^3+5x^2-12x)=0$	1M	for $(px+q)(x^3+5x^2-12x)=0$
$x(3x+4)(x^2+5x-12)=0$		
$x=0$, $3x+4=0$ or $x^2+5x-12=0$	1M	
$x=0$, $x=\frac{-4}{3}$ or $x=\frac{-5\pm\sqrt{73}}{2}$	1M	
Note that both $\frac{-5+\sqrt{73}}{2}$ and $\frac{-5-\sqrt{73}}{2}$ are not rational numbers.		
Also note that both $\frac{-4}{3}$ and 0 are rational roots of $h(x)=0$.		
Thus, the equation $h(x)=0$ has 2 rational roots.	1A	f.t.
	----------(4)	
14. (a) Let l cm be the slant height of the circular cone.		
$\pi(14)(l)=700\pi$	1M	
$l=50$		
The height of the circular cone		
$=\sqrt{l^2-14^2}$	1M	
$=\sqrt{50^2-14^2}$		
$=48$ cm	1A	
	----------(3)	
(b) (i) The volume of Y		
$=\frac{1}{3}\pi(14^2)(48)\left(1-\left(\frac{1}{\sqrt{15+1}}\right)^3\right)$	1M+1M	
$=3\,087\pi\ \text{cm}^3$	1A	
(ii) Let d cm be the diameter of each sphere.		
$\frac{4}{3}\pi\left(\frac{d}{2}\right)^3=\frac{3\,087\pi}{2}$	1M	
$d=21$	1A	
Thus, the diameter of each sphere is 21 cm .		
	----------(5)	

Solution	Marks	Remarks
15. (a) The required probability		
$=\dfrac{C_2^5}{C_2^9}$	1M	for denominator
$=\dfrac{5}{18}$	1A	r.t. 0.278
	----------(2)	
(b) The required probability		
$=\dfrac{5}{18}+\dfrac{C_1^5C_1^4C_3^9+C_2^4C_3^8}{C_2^9C_3^{10}}$	1M	for (a) $+p$, where $0<p<1$
$=\dfrac{67}{90}$	1A	r.t. 0.744
	----------(2)	
16. (a) Note that $p+5p=-a$ and $p(5p)=b$.	1M	for either one
Therefore, we have $6p=-a$ and $5p^2=b$.		
So, we have $5\left(\dfrac{-a}{6}\right)^2=b$.		
Thus, we have $5a^2=36b$.	1	
	----------(2)	
(b) Let t be the x-coordinate of Q .		
Then, the x-coordinate of R is $5t$.		
Putting $y=mx$ in $x^2+y^2-6x-12y+20=0$, we have		
$x^2+(mx)^2-6x-12(mx)+20=0$	1M	
So, we have $(m^2+1)x^2-(12m+6)x+20=0$.		
Therefore, t and $5t$ are the roots of the equation		
$x^2-\dfrac{6(2m+1)}{m^2+1}x+\dfrac{20}{m^2+1}=0$		
By (a), we have $5\left(\dfrac{-6(2m+1)}{m^2+1}\right)^2=36\left(\dfrac{20}{m^2+1}\right)$.	1M	
Simplifying, we have $(2m+1)^2=4(m^2+1)$.		
Therefore, we have $4m=3$.		
Solving, we have $m=\dfrac{3}{4}$.	1A	0.75
	----------(3)	

Solution	Marks	Remarks
17. (a) By sine formula, we have		
$\dfrac{\sin \angle XWY}{XY} = \dfrac{\sin \angle WYX}{WX}$	1M	
$\dfrac{\sin \angle XWY}{5} = \dfrac{\sin 70^\circ}{6}$		
$\angle XWY \approx 51.54318937^\circ$ or $\angle XWY \approx 128.4568106^\circ$ (rejected)		
$\angle XWY \approx 51.5^\circ$	1A	r.t. 51.5°
	----------(2)	
(b) Let P be the projection of Z on the triangle WXY.		
Note that $PW = PX = PY$.		
Denote the mid-point of XY by M.		
The angle between the triangles WXY and XYZ is $\angle PMZ$.	1M	
The centre of the circle which passes through W, X and Y is P.		
So, we have $\angle MPX = \dfrac{1}{2}\angle XPY = \angle XWY$.		
MP		
$= PX \cos \angle MPX$	1M	
$= PW \cos \angle XWY$		
$\tan \angle PWZ = \dfrac{PZ}{PW}$	1M	either one
$\tan 30^\circ = \dfrac{PZ}{PW}$		
$PZ = PW \tan 30^\circ$		
$\tan \angle PMZ$		
$= \dfrac{PZ}{MP}$		
$= \dfrac{PW \tan 30^\circ}{PW \cos \angle XWY}$		
$= \dfrac{\tan 30^\circ}{\cos \angle XWY}$		
$\approx \dfrac{\tan 30^\circ}{\cos 51.54318937^\circ}$ (by (a))		
≈ 0.928328501		
Therefore, we have $\tan \angle PMZ < 1$.		
Hence, we have $\angle PMZ < 45^\circ$.		
Thus, the angle between the triangles WXY and XYZ does not exceed 45° .	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
18. (a) $\frac{\beta}{7}=\frac{7}{\alpha}$	1M	
$\alpha\beta = 49$		
$\log_7 \alpha\beta = \log_7 49$		
$\log_7 \alpha + \log_7 \beta = 2$	1M	
$\log_7 \alpha = 2 - \log_7 \beta$	1A	
	----------(3)	
(b) $\log_\alpha \beta - \log_7 \beta = \log_7 \beta - \log_\beta \alpha$	1M	
$\frac{\log_7 \beta}{\log_7 \alpha} - \log_7 \beta = \log_7 \beta - \frac{\log_7 \alpha}{\log_7 \beta}$	1M	
Let $u = \log_7 \beta$.		
$\frac{u}{2-u} - u = u - \frac{2-u}{u}$ (by (a))	1M	for using the result of (a)
$\frac{u^2-u}{2-u} = \frac{u^2+u-2}{u}$		
$\frac{u(u-1)}{2-u} = \frac{(u+2)(u-1)}{u}$		
$u^2 = (u+2)(2-u)$ (since $u \neq 1$)		
$u^2 = 2$		
$u = \sqrt{2}$ or $u = -\sqrt{2}$ (rejected)		
$\log_7 \beta = \sqrt{2}$		
The common difference of the arithmetic sequence		
$= \log_7 \beta - \log_\beta \alpha$		
$= \log_7 \beta - \frac{\log_7 \alpha}{\log_7 \beta}$		
$= \log_7 \beta - \frac{2 - \log_7 \beta}{\log_7 \beta}$		
$= \sqrt{2} - \frac{2-\sqrt{2}}{\sqrt{2}}$	1M	
$= 1$	1A	
	----------(5)	

Solution	Marks	Remarks
19. (a) The slope of PQ $=\dfrac{t-0}{32-50}$ $=\dfrac{-t}{18}$		
Note that the x-coordinate of G is 25 .	1M	
The equation of the perpendicular bisector of PR is $y-t=\dfrac{18}{t}(x-32)$	1M	either one
Putting $x=25$ in $y-t=\dfrac{18}{t}(x-32)$, we have $y=\dfrac{t^2-126}{t}$.		
Therefore, the coordinates of G are $\left(25,\dfrac{t^2-126}{t}\right)$.	1A	
Also note that the x-coordinate of R is 14 .	1M	
The equation of the straight line which passes through O and is perpendicular to PR is $y=\dfrac{18}{t}x$.		
Putting $x=14$ in $y=\dfrac{18}{t}x$, we have $y=\dfrac{252}{t}$.		
Thus, the coordinates of H are $\left(14,\dfrac{252}{t}\right)$.	1A	
	----------(5)	
(b) (i) As $\angle PQS=\angle POQ$, we have $\tan\angle PQS=\tan\angle POQ$.		
$\dfrac{50-32}{t}=\dfrac{t}{32}$	1M	for either side
$\dfrac{18}{t}=\dfrac{t}{32}$ $t^2-576=0$		
Since $t>0$, we have $t=24$.	1	
(ii) By (a), the coordinates of G are $\left(25,\dfrac{75}{4}\right)$.	1M	for using the result of (a)
The coordinates of Q are $(32,24)$.		
The slope of OG $=\dfrac{\frac{75}{4}-0}{25-0}$ $=\dfrac{3}{4}$		
The slope of OQ $=\dfrac{24-0}{32-0}$ $=\dfrac{3}{4}$		
Therefore, the slope of OG and the slope of OQ are equal. Thus, O , G and Q are collinear.	1A	f.t.

Solution	Marks	Remarks
Since OQ is a median of ΔOPR, Q is the mid-point of PR. Note that G is the circumcentre of ΔOPR. Therefore, we have $GQ \perp PR$.	1M	either one
$\angle OQP$ $= 180° - \angle POQ - \angle OPQ$ $= 180° - \angle PQS - \angle OPQ$ $= \angle PSQ$ $= 90°$ So, we have $OQ \perp PR$.		
Hence, we have $GQ // OQ$. Thus, O, G and Q are collinear.	1A	f.t.
(iii) Note that OQ is perpendicular to PR and $\Delta OPQ \cong \Delta ORQ$. Hence, OQ is the angle bisector of $\angle POR$. Therefore, I lies on OQ. Denote the foot of the perpendicular from I to OP by J. Then, we have $\Delta OIJ \sim \Delta OPQ$. Let r be the radius of the inscribed circle of ΔOPR. So, we have $\frac{OQ-r}{r} = \frac{OP}{PQ}$.	1M	
Since $OQ = 40$ and $PQ = 30$, we have $\frac{40-r}{r} = \frac{50}{30}$. Solving, we have $r = 15$. Hence, the coordinates of I are $(20, 15)$. Also note that the coordinates of H are $\left(14, \frac{21}{2}\right)$. Further note that O, H, I, G and Q are collinear. Since OQ is a median of ΔOPR, we have $PQ = QR$. The required ratio $= \frac{1}{2}(GH)(QR) : \frac{1}{2}(IQ)(PQ)$ $= GH : IQ$		
$= (25-14) : (32-20)$	1M	
$= 11 : 12$	1A	
	----------(7)	

Paper 2

Question No.	Key	Question No.	Key
1.	C (78)	26.	B (40)
2.	C (79)	27.	C (51)
3.	A (72)	28.	D (65)
4.	B (87)	29.	C (75)
5.	A (70)	30.	A (84)
6.	D (77)	31.	B (62)
7.	C (82)	32.	D (64)
8.	D (74)	33.	A (39)
9.	A (43)	34.	B (63)
10.	D (67)	35.	D (32)
11.	B (81)	36.	B (40)
12.	D (39)	37.	D (25)
13.	B (66)	38.	C (50)
14.	B (62)	39.	A (31)
15.	A (50)	40.	A (29)
16.	D (27)	41.	C (33)
17.	B (44)	42.	B (45)
18.	A (59)	43.	C (40)
19.	C (54)	44.	D (60)
20.	A (52)	45.	A (47)
21.	C (34)		
22.	B (42)		
23.	D (38)		
24.	C (58)		
25.	A (56)		

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.